3.4. Solved Problems - Clutch Drive System Design

Problem 1

A plate clutch consists of a flat driven plate gripped between the driving plate and the pressure plate so that two active driving surfaces each having an inner diameter of 204 mm and an outer diameter of 356 mm. The coefficient of friction is 0.4. The working pressure is limited to 172 kN/m^2.

(a) Assuming that the pressure is uniform, determine the power which can be transmitted at a rotational speed of 1000 rev/min.

(b) If the clutch is worn so that the intensity of pressure is inversely proportional to the radius, the total axial force of the pressure plate remaining unaltered, calculate the power which can be transmitted at 1000 rev/min, and the greatest intensity of pressure on the friction surfaces.

Solution

(a) Uniform Pressure - Transmitted Power:

Data:

p = 172 kNm^{-2} = 172 x 10^3 Nm^{-2}

r_1 = 356/2 mm = 0.178 m

r_2 = 204/2 mm = 0.102 m

∴ R = 140 mm = 0.14 m

n = 2

Axial thrust force, $W = \pi.p\left(r_1^2 - r_2^2\right) = \pi x(172x10^3)(0.178^2 - 0.102^2)$ = 11.49 kN

Torque, $T = \frac{2}{3}.\mu.W\frac{\left(r_1^3 - r_2^3\right)}{\left(r_1^2 - r_2^2\right)}xn = \frac{2}{3}x0.4x(11.49x10^3)\frac{\left(0.178^3 - 0.102^3\right)}{\left(0.178^2 - 0.102^2\right)}x2$ = 1318 Nm

Power, P = Tω = $\frac{2\pi NT}{60} = \frac{2\pi x1000x1318}{60}$ = **138 kW**

(b) Uniform Wear – Transmitted Power:

T = μWR.n = 0.4(11.49 x 10^3)(0.14)2 = 1287 Nm

Power, P = Tω = $\frac{2\pi NT}{60} = \frac{2\pi x1000x1287}{60}$ = **135 kW**

Axial thrust force, $W = 2\pi.c\left(r_1 - r_2\right) = 2\pi.p.r\left(r_1 - r_2\right)$ where p occurs at the smallest radius, i.e r_2.

∴ $p = \frac{W}{2\pi r_2(r_1 - r_2)} = \frac{11.49x10^3}{2\pi x0.102(0.178 - 0.102)}$ = **236 kNm^{-2}**

Problem 2

A multiple disc clutch consisting of 5 input discs and 4 output discs is shown in Figure 3.4. This clutch is required to transmit a maximum power of 2.7 kW at 1600 rpm, and the inner diameter of the clutch is restricted to 55 mm due to the size of the shaft. Making the assumption of:

a. ***Uniform pressure***

b. ***Uniform wear***

Determine the required outside diameter of the discs and the required axial force. For both cases the coefficient of friction can be assumed to be 0.12, and the pressure is not to exceed 360 kNm^{-2}.

Solution

(a) Uniform Pressure – Outside Diameter and Required Axial Force:

Data:

p = 360 kNm^{-2} = 360 x 10^{3} Nm^{-2}

r_1 = ?

r_2 = 55/2 mm = 0.0275 m

μ = 0.12

n = 8

P = 2.7 kW = 2.7 x 10^{3} W

N = 1600 rpm

Total Torque, $T_t = \dfrac{60 \times P}{2\pi N} = \dfrac{60 \times 2700}{2\pi \times 1600}$ = 16.11 Nm

Torque per pair of contact plates, $T = T_t / n = 16.11/8$ = 2.014 Nm

As torque, $T = \dfrac{2}{3}.\pi\mu.p\left(r_1^3 - r_2^3\right)$ Nm

$$\Rightarrow 2.014 = \frac{2}{3}.\pi \times 0.12 \times 360x10^3\left(r_1^3 - (0.0275)^3\right)$$

$$\Rightarrow r_1^3 - (0.0275)^3 = \frac{3}{2\pi} \times \frac{2.014}{0.12 \times 360x10^3}$$

$$\Rightarrow r_1^3 = \frac{3}{2\pi} \times \frac{2.014}{0.12 \times 360x10^3} + (0.0275)^3$$

$$=> \Rightarrow r_1^3 = 4.3060x10^{-5}$$

$$\Rightarrow r_1 = 0.0351m$$

$$= 35.05mm$$

$$\Rightarrow d_1 = 70.1mm$$

Therefore, Axial thrust force, $W = \pi.p\left(r_1^2 - r_2^2\right) = \pi x(360x10^3)(0.03505^2 - 0.0275^2)$

= 534.12 N

(b) Uniform Wear – Outside Diameter and Required Axial Force

$$T = \mu WR. = \mu\pi p\left(r_1^2 - r_2^2\right)\left(\frac{r_1 + r_2}{2}\right) \text{ Nm}$$

$$\Rightarrow 2.014 = 0.12\pi \times 360x10^3\left(r_1^2 - (0.0275)^2\right)\left(\frac{r_1 + 0.0275}{2}\right)$$

$$\Rightarrow 2.014 = 135{,}716.80\left(r_1^2 - (0.0275)^2\right)\left(\frac{r_1}{2} + 0.01375\right)$$

$$\Rightarrow 1.4840x10^{-5} = \left(r_1^2 - (0.0275)^2\right)\left(\frac{r_1}{2} + 0.01375\right)$$

This equation is difficult to solve, so the value is more easily obtained by assuming that r_1 is equal to 35 mm, from part a), then equating the two sides of the equation:

$$RHS = \left(r_1^2 - (0.0275)^2\right)\left(\frac{r_1}{2} + 0.01375\right)$$
$$= \left(0.035 - (0.0275)^2\right)\left(\frac{0.035}{2} + 0.01375\right)$$
$$= 1.4648x10^{-5}$$

This is approximately equal to the LHS, but would ideally be within 1%, so:

$$\%diff = \frac{1.4840 - 1.4648}{1.4840} \times 100$$
$$= 1.29\%$$

Therefore try r_1 = 35.1 mm;

$$RHS = \left(r_1^2 - (0.0275)^2\right)\left(\frac{r_1}{2} + 0.01375\right)$$
$$= \left(0.0351 - (0.0275)^2\right)\left(\frac{0.0351}{2} + 0.01375\right)$$
$$= 1.4891x10^{-5}$$

This is approximately equal to the LHS, but would ideally be within 1%, so:

$$\%diff = \frac{1.4891 - 1.4840}{1.4840} \times 100$$
$$= 0.35\%$$

Therefore d_1 = 70.2 mm, not greatly different from part a).

Therefore, Axial thrust force, $W = \pi.p\left(r_1^2 - r_2^2\right) = \pi x(360x10^3)(0.0351^2 - 0.0275^2)$
= 538.07 N

Problem 3

A cone clutch is designed to transmit 210 Nm of torque at 1220 rpm. The clutch contact surfaces have an outer diameter of 360 mm and the semi-cone angle is 6.5°. Assuming the face width is 70 mm and the coefficient of friction is 0.22, determine the axial force required to transmit the torque assuming:

a. Uniform pressure

b. Uniform wear

Solution

Data:

T = 210 Nm

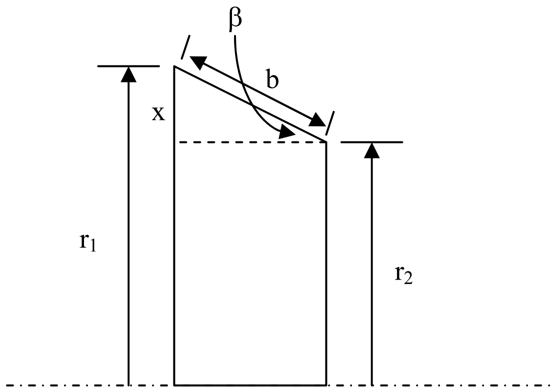
D_1 = 360 mm = 0.36 m

r_1 = 0.36/2 m = 0.18 m

$\beta = 6.5^0$

b = 70 mm

μ = 0.22



From the geometry:

$$r_2 = r_1 - x \qquad (1)$$

$$\sin\beta = \frac{x}{b}$$

$$\Rightarrow x = b\sin\beta \qquad (2)$$

Substituting (2) into (1):

$$\Rightarrow r_2 = r_1 - b\sin\beta$$
$$= 0.18 - 0.07\sin 6.5 = 0.1721m$$

a) Assuming uniform pressure:

$$T = \frac{2}{3} \cdot \frac{\mu . W}{\sin \beta} \frac{\left(r_1^3 - r_2^3\right)}{\left(r_1^2 - r_2^2\right)}$$

$$\Rightarrow 210 = \frac{2}{3} \cdot \frac{0.22.W}{\sin 6.5} \frac{\left(0.18^3 - 0.1721^3\right)}{\left(0.18^2 - 0.1721^2\right)}$$

$$\Rightarrow W = 210 \times \frac{3}{2} \times \frac{\sin 6.5}{0.22} \times \frac{\left(0.18^2 - 0.1721^2\right)}{\left(0.18^3 - 0.1721^3\right)}$$

$$\Rightarrow W = 615.53N$$

b) Assuming uniform wear:

$$T = \frac{\mu . WR}{\sin \beta} = \frac{\mu . W}{\sin \beta} \times \frac{r_1 + r_2}{2}$$

$$\Rightarrow 210 = \frac{0.22.W}{\sin 6.5} \times \frac{0.18 + 0.1721}{2}$$

$$\Rightarrow W = \frac{210 \times 2 \sin 6.5}{0.22 \times \left(0.18 + 0.1721\right)}$$

$$\Rightarrow W = 613.79N$$

Problem 4

A plate clutch has 5 driving discs and 4 driven discs, i.e. 8 pairs of contact surfaces, each of 185 mm external diameter and 135 mm internal diameter.

(a) Assuming uniform pressure, calculate the total axial spring load required to permit the clutch to transmit 38 kW at 1470 rev/min if the coefficient of friction between the disc surfaces is 0.32.

(b) If there are 8 springs, each of stiffness 13 kNm^{-1}, and each of the contact surfaces has lost 0.145 mm due to wear, calculate the maximum power which can be transmitted under the above conditions:

Solution

Data:

P = 38 kW

N = 1470 rpm

D_1 = 185 mm = 0.185 m

r_1 = 0.185/2 m = 0.0925 m

D_2 = 135 mm = 0.135 m

r_2 = 0.135/2 m = 0.0675 m

μ = 0.32

n = 8

k = 13 kNm^{-1} = 13 x 10^3 Nm^{-1}

Wear per contact surface = 0.145 mm

(a) Total Torque, $T_t = \dfrac{60 \times P}{2\pi N} = \dfrac{60 \times 38x10^3}{2\pi \times 1470}$ = 246.85 Nm

Torque per pair of contact plates, $T = T_t / n = 246.85 / 8$ = 30.86 Nm

As torque, $T = \dfrac{2}{3}.\mu.W \dfrac{\left(r_1^3 - r_2^3\right)}{\left(r_1^2 - r_2^2\right)}$ Nm

$$\Rightarrow 30.86 = \frac{2}{3}.0.32.W \frac{\left(0.0925^3 - 0.0675^3\right)}{\left(0.0925^2 - 0.0675^2\right)}$$

$$\Rightarrow W = \frac{3}{2}.\frac{1}{0.32}.30.86 \frac{\left(0.0925^2 - 0.0675^2\right)}{\left(0.0925^3 - 0.0675^3\right)} = 1195.60N$$

(b) W_{worn} = W – F, where F = k_T.x

k_T (8 springs in parallel) = 8 x 13 x 10^3 = 104 x 10^3 Nm^{-1}

x (8 contact pairs x two surfaces per pair x wear per contact surface) = 8 x 2 x 0.145 mm

= 2.32 mm

Therefore:

W_{worn} = 1195.60 – 104 x 10^3 x 2.32 x 10^{-3}

= 1195.60 – 241.28

= 954.32 N

Therefore:

$$
\begin{aligned}
T &= \mu WRxn \\
&= 0.32x954.32x\frac{(0.0925 + 0.0675)}{2}x8 \\
&= 195.45Nm
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
P &= Tx\omega \\
&= 195.45x\frac{2x\pi x1470}{60} \\
&= 30{,}086.54W \\
&= 30.09kW
\end{aligned}
$$